Vorticity (Ch. 4.2)

Vorticity: microscopic measure of rotation in a fluid (curl of wind field, vector)

velocity in a fixed frame of reference

$$\overrightarrow{U_a} = \overrightarrow{U_e} + \vec{U} = \square$$

absolute vorticity

$$\overrightarrow{\omega_a} \equiv \square$$

relative vorticity

$$\vec{\omega} \equiv \square$$

planetary vorticity

$$\overrightarrow{\omega_e} \equiv \square = \square$$

Vertical component of vorticity

planetary vorticity

$$\hat{k} \cdot \square = 2\Omega \sin \emptyset = f$$

relative vorticity (zeta)

$$\zeta \equiv \hat{k} \cdot \vec{\omega} = \square$$

absolute vorticity (eta)

$$\eta \equiv \hat{k} \cdot \overrightarrow{\omega_a} = \square$$

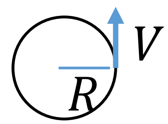
**can absolute vorticity be negative?*

Circulation (Ch. 4.1)

Circulation: macroscopic measure of rotation in a fluid (for a finite area, scalar)

line integral of the velocity fields about a closed contour in a fluid

$C =$ [ ]



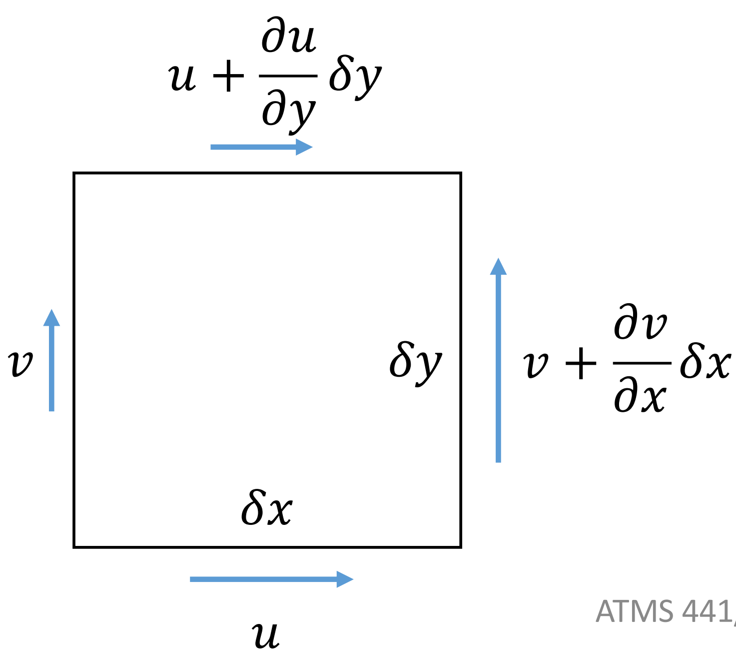
**circulation about the circle:* [ ]

using the Stokes's theorem

$C =$ [ ] $=$ [ ]

**relationship between circulation and vorticity*

Vorticity and circulation



circulation around the box

$\delta C =$ [ ]

[ ]

$\delta C =$ [ ]

$\zeta =$ [ ]

**relationship between circulation and vorticity*

Vorticity equation (Ch. 4.3)

Cartesian coordinate form

horizontal momentum equation

$$Eq.(1): \frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad Eq.(2): \frac{Dv}{Dt} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

d(2)/dx – d(1)/dy

$$\frac{D}{Dt}(\zeta + f) =$$

A B C D

- A:
- B:
- C:
- D:

Vorticity equation (Ch. 4.3)

Scale analysis of the vorticity equation

vorticity equation

$$\frac{D}{Dt}(\zeta+f) = -(\zeta+f)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

$$\frac{D}{Dt}(\zeta+f) =$$

$$=$$

$$=$$

$$f = f_0 + \beta y$$
$$\beta = df/dy$$
$$f_0 \sim 10^{-4} s^{-1}$$
$$\beta \sim 10^{-11} s^{-1} m^{-1}$$

$$|\mathrm{u},\mathrm{v}| \sim 10\ m\ s^{-1}$$
$$|\mathrm{w}| \sim 10^{-2}\ m\ s^{-1}$$
$$|\delta \mathrm{x}, \delta y| \sim 10^{6}\ m$$
$$|\delta z| \sim 10^{4}\ m$$
$$|\delta p| \sim 10^{1}\ hPa$$
$$|\rho| \sim 10^{-1} kg\ m^{-3}$$
$$|\delta\rho/\rho| \sim 10^{-2}$$
$$|\delta t| \sim |\delta x/u| \sim 10^{5}\ s$$

$$g = 9.81\ m\ s^{-2}$$

Vorticity equation (Ch. 4.3)

Scale analysis of the vorticity equation

vorticity equation

$$\frac{D}{Dt}(\zeta + f) = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

$$f = f_0 + \beta y$$
$$\beta = df/dy$$
$$f_0 \sim 10^{-4} s^{-1}$$
$$\beta \sim 10^{-11} s^{-1} m^{-1}$$

$$|\mathrm{u}, \mathrm{v}| \sim 10\ m\ s^{-1}$$
$$|\mathrm{w}| \sim 10^{-2}\ m\ s^{-1}$$
$$|\delta \mathrm{x}, \delta y| \sim 10^{6}\ m$$
$$|\delta z| \sim 10^{4}\ m$$
$$|\delta p| \sim 10^{1}\ hPa$$
$$|\rho| \sim 10^{-1} kg\ m^{-3}$$
$$|\delta\rho/\rho| \sim 10^{-2}$$
$$|\delta t| \sim |\delta x/u| \sim 10^{5}\ s$$

$$g = 9.81\ m\ s^{-2}$$

vortex stretching term

$$(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) =$$

tilting term

$$\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right)$$

solenoidal term

$$\frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

Vorticity equation (Ch. 4.3)

Scale analysis of the vorticity equation

vorticity equation for synoptic scale, midlatitude motinos

$$\frac{D_h}{Dt}(\zeta + f) = \boxed{} \qquad \frac{D_h}{Dt} = \boxed{}$$

for intense cyclones (relative vorticity is large)

$$\frac{D_h}{Dt}(\zeta + f) = \boxed{}$$

cyclones - convergence

anticyclones - divergence

planetary vorticity as a function of latitude

$$f(10^oN) \cong \boxed{}\, s^{-1}$$
$$f(30^oN) \cong \boxed{}\, s^{-1}$$
$$f(50^oN) \cong \boxed{}\, s^{-1}$$
$$f(70^oN) \cong \boxed{}\, s^{-1}$$
$$f(90^oN) \cong \boxed{}\, s^{-1}$$

Potential vorticity (Ch. 4.4)

Kevin's circulation theorem

Circulation theorem (Eq. 4.3, Ch. 4.1)

$$\frac{DC_a}{Dt} = -\oint \frac{1}{\rho} dp$$

**Only the solenoidal term can change circulation (the divergence and tilting term do not affect circulation)*

If density is a function only of pressure (i.e. a barotropic fluid)

$$\frac{DC_a}{Dt} = \square$$

In an isentropic (constant θ) coordinate

the first law of thermodynamics

$$c_p \frac{D \ln \theta}{Dt} = \frac{J}{T}$$

**if J=0, air parcel will stay at the constant θ level*

potential temperature

$$\theta = T \left(\frac{p_s}{p}\right)^{\frac{R}{c_p}}$$

using ideal gas law

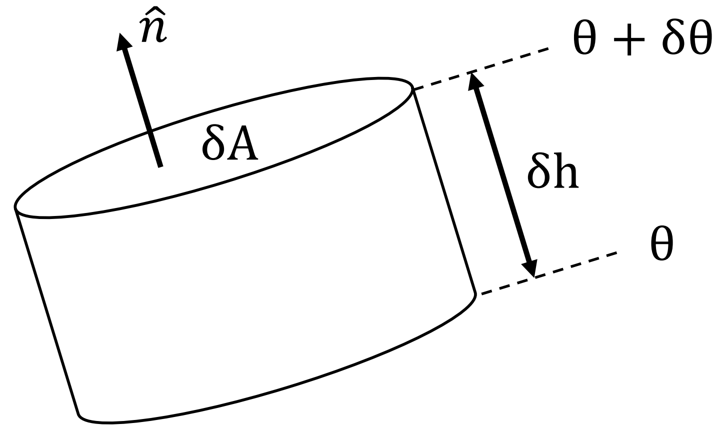
$$\rho = \square$$

circulation will be conserved following the motion

$$\frac{DC_a}{Dt} = \square \qquad C_a = \iint \square \, dA$$

Potential vorticity (Ch. 4.4)

In an isentropic (constant θ) coordinate



circulation about the perimeter

$$C_a = \iint \overrightarrow{\omega_a} \cdot \hat{n}\, dA$$

$$= \boxed{\quad}$$

$$=$$

$$=$$

mass of the air parcel

$$\delta m = \rho \delta A \delta h$$

$$\delta A = \frac{\delta m}{\rho \delta h} = \frac{\delta m}{\rho}\frac{|\nabla\theta|}{\delta\theta}$$

distance between two surfaces

$$\delta\theta \approx |\nabla\theta| \delta h$$

$$\delta h = \frac{\delta\theta}{|\nabla\theta|}$$

from the conservation of circulation

$$\frac{DC_a}{Dt} = \frac{D}{Dt}\boxed{\quad} = 0$$

unit normal vector

$$\hat{n} = \frac{\nabla\theta}{|\nabla\theta|}$$

drop constant fields

$$\frac{D}{Dt}\boxed{\quad} = 0$$

Ertel's potential vorticity: conserved following motion

$$\Pi \equiv \boxed{\quad}$$

**momentum field and thermodynamic field are not independent*

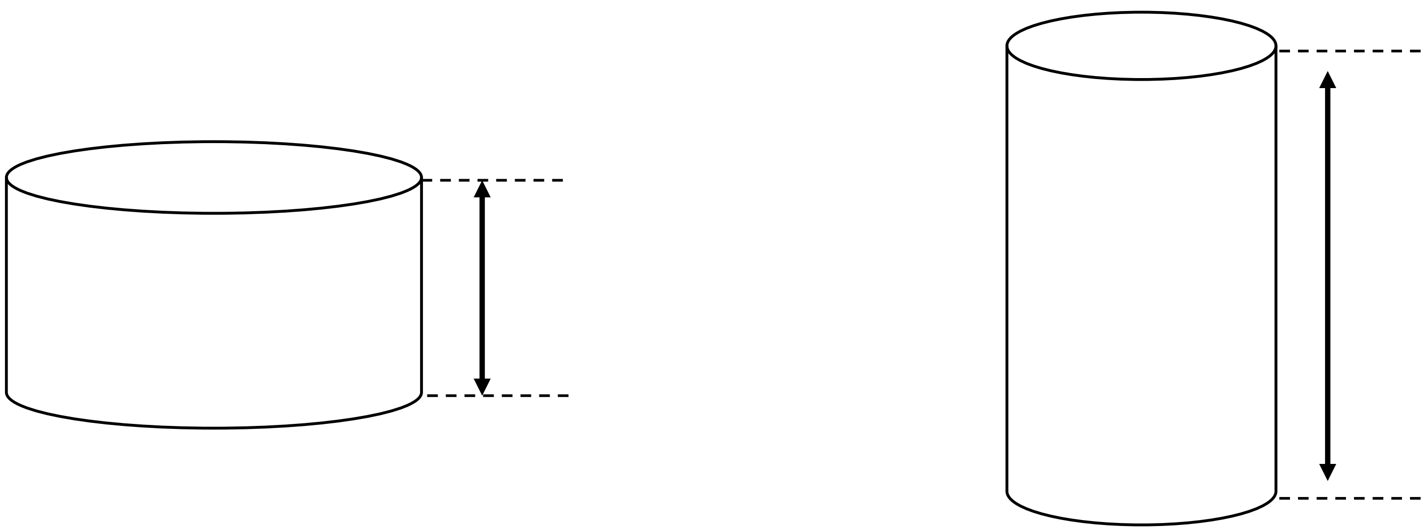
Potential vorticity (Ch. 4.4)

Potential vorticity

vertical component of the Ertel's potential vorticity

**absolute vorticity and static stability is not independent*

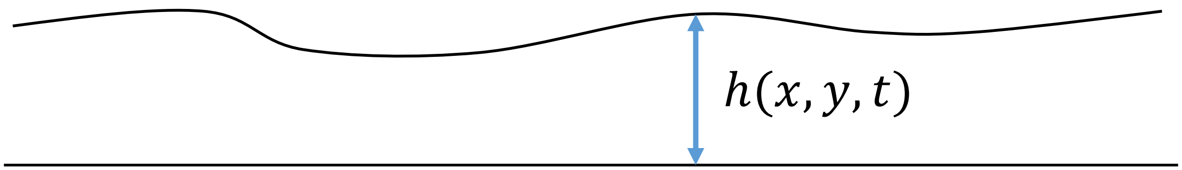
**this component is larger than horizontal component (why?)*



Shallow water equations (Ch. 4.5)

Shallow water

homogeneous, incompressible fluid (constant density)



hydrostatic approximation

$$\frac{\partial p}{\partial z} = \qquad\qquad \delta p =$$

integrate from z=h to z=z

$$p(z) =$$

horizontal pressure gradient

$$\frac{\partial p}{\partial x} = \qquad\qquad \frac{\partial p}{\partial y} =$$

horizontal momentum equation

$$\frac{D_h u}{Dt} - fv = \qquad\qquad \frac{Dv}{Dt} + fu =$$

Shallow water equations (Ch. 4.5)

Shallow water

continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \delta w = \boxed{}$$

integrate from z=0 to z=h

$$w(z = h) = \frac{D_h h}{Dt} = \boxed{}$$

shallow water equations

$$\boxed{} \quad \boxed{} \quad \boxed{}$$

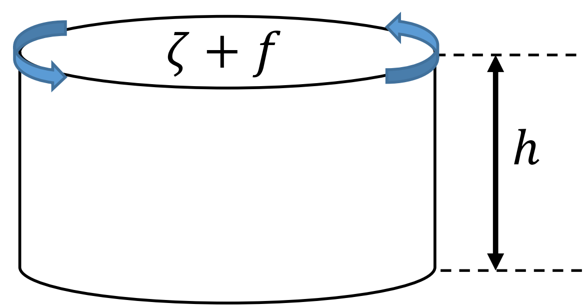
shallow water vorticity equation

$$\frac{D_h}{Dt}(\zeta + f) = \boxed{}$$

combine with continuity equation

$$\frac{D_h}{Dt}\boxed{} = 0$$

*what is conserved?



*what could change relative vorticity?

Shallow water equations (Ch. 4.5)

Barotropic potential vorticity

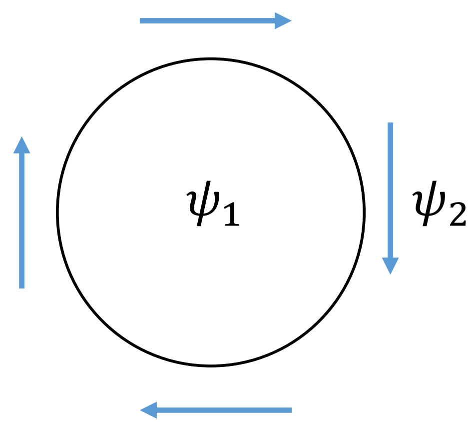
if h=constant

$$\frac{D_h}{Dt}\boxed{\quad} = 0$$

*define streamfunction (for nondivergent horizontal motion, *check divergence of horizontal wind)*

$$u = \boxed{\quad} \qquad v = \boxed{\quad}$$

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \boxed{\quad}$$



$\psi_1 > \psi_2$

vorticity equation becomes

$$\frac{D_h}{Dt}\boxed{\quad} = 0$$

$$\frac{\partial}{\partial t}\nabla^2\psi = \boxed{\quad}$$

**how many dependent variables?*

**can this equation be used to describe evolution of flow?*